

DOUBLY TRANSITIVE GROUPS AND CYCLIC QUANDLES

LEANDRO VENDRAMIN

ABSTRACT. We prove that for $n > 2$ there exists a quandle of cyclic type of size n if and only if n is a power of a prime number. This establishes a conjecture of S. Kamada, H. Tamaru and K. Wada. As a corollary, every finite quandle of cyclic type is an Alexander quandle. We also prove that finite doubly transitive quandles are of cyclic type. This establishes a conjecture of H. Tamaru.

INTRODUCTION

Quandles are algebraic structures deeply related to the Reidemeister moves of classical knots. These structures play an important role in knot theory because they produce strong knot invariants, see for example [5], [6] and [7]. The applications of quandles in knot theory force us to study certain particular classes of quandles. One of these classes is the class of finite quandles of cyclic type. The idea of studying such quandles goes as far as [13]. Quandles of cyclic type were independently considered in [9] and [18].

In this note we present the proofs of two conjectures related to quandles of cyclic type. First we prove the following theorem, conjectured by S. Kamada, H. Tamaru and K. Wada, see [12, Conjecture 4.7].

Theorem 1. *Let $n \geq 3$. Then there exists a quandle of size n of cyclic type if and only if n is a power of a prime number.*

K. Wada independently proved that cyclic quandles with a prime power size are Alexander quandles [19]. Theorem 1 yields the following stronger result.

Corollary 2. *Let X be a finite quandle of cyclic type. Then $|X|$ is a power of a prime number and X is an Alexander simple quandle over the field with $|X|$ elements.*

Finally, using the classification of doubly transitive finite groups with simple socle [3] we prove the following theorem.

Theorem 3. *Every finite doubly transitive quandle is an Alexander simple quandle.*

The theorem gains in interest if we know that doubly transitive Alexander quandles are of cyclic type. This was proved by K. Wada [19]. Then one immediately obtains the following corollary, which proves a conjecture of H. Tamaru, see [18, Conjecture 5.1].

Corollary 4. *Every finite doubly transitive quandle is of cyclic type.*

The principal significance of the corollary is that it advances the classification of k -transitive quandles for $k \geq 2$. On the other hand, the classification of finite indecomposable quandles is somewhat out of reach. Thus the following seems to be an interesting problem.

Problem 5. *Classify finite primitive quandles.*

The paper is organized as follows. In Section 1 we set up notations and terminology, and we review some basic facts about quandles and permutation groups. Section 2 is devoted to prove Theorem 1 and Corollary 2. The proof of the theorem is based on the following observation: the inner group of a finite quandle of cyclic type is a Frobenius group. The proof of the corollary uses Theorem 1 and the classification of simple quandles of Andruskiewitsch and Graña [2, §3]. In Section 3 we prove Theorem 3. The proof depends on the classification of doubly transitive groups with simple socle.

1. PRELIMINARIES

Recall that a *quandle* is a set X with a binary operation $\triangleright: X \times X \rightarrow X$ such that $x \triangleright x = x$ for all $x \in X$, the map $\varphi_x: X \rightarrow X$, $y \mapsto x \triangleright y$, is bijective for all $x \in X$, and $x \triangleright (y \triangleright z) = (x \triangleright y) \triangleright (x \triangleright z)$ for all $x, y, z \in X$. The *inner group* of X is the group $\text{Inn}(X) = \langle \varphi_x \mid x \in X \rangle$. The quandle X is *indecomposable* (or *connected*) if $\text{Inn}(X)$ acts transitively on X .

Lemma 6. *Let X be a quandle and $x \in X$. Then φ_x is a central element of the stabilizer of x in $\text{Inn}(X)$.*

A quandle X is *primitive* if $\text{Inn}(X)$ acts primitively on X . For $k \geq 1$ we say that X is *k-transitive* if $\text{Inn}(X)$ acts k -transitively on X . It is worth pointing out that 1-transitive means indecomposable, and that 2-transitive (or *doubly transitive*) quandles are called two-point homogeneous in [18]. A similar argument to that of [20, Thm. 9.6] shows that doubly transitive quandles are primitive. Similarly, $(k+1)$ -transitive quandles are k -transitive for all $k \geq 1$. The following result of McCarron [15, Prop. 5] shows that higher transitivity is a rare phenomenon: the dihedral quandle with three elements is the unique 3-transitive quandle.

Lemma 7 (McCarron). *Let $k \in \mathbb{N}$ with $k \geq 2$ and X be a finite k -transitive quandle with at least four elements. Then $k \leq 2$.*

Proof. Suppose that $k \geq 3$. Since X is k -transitive, it is indecomposable and nontrivial. Thus let $x, y \in X$ such that $|\{x, y, x \triangleright y\}| = 3$. By assumption, there exists $z \in X \setminus \{x, y, x \triangleright y\}$. Since $\text{Inn}(X)$ acts k -transitively on X and $k \geq 3$, there exists $f \in \text{Inn}(X)$ such that $f(x) = x$, $f(y) = y$ and $f(x \triangleright y) = z$. Then $x \triangleright y = f(x) \triangleright f(y) = f(x \triangleright y) = z$, a contradiction. \square

We shall also need the following lemma of [14]. Recall that a quandle is *simple* if it has no quotients except itself and the trivial quandle of one element [11].

Lemma 8 (McCarron). *Let X be a finite quandle and suppose that $\text{Inn}(X)$ acts primitively on X . Then X is simple.*

Proof. Suppose that X is not simple. Then there exist a nontrivial quandle $Q \neq X$ and a surjective homomorphism of quandles $p: X \rightarrow Q$. Consider the equivalence relation over X given by $x \equiv y$ if and only if $p(x) = p(y)$. We claim that the orbits of this action form a system of blocks for G . To prove our claim let $x \in X$ and $\Delta_x = \{y \in X \mid p(y) = p(x)\}$ be an equivalence class. Then $\varphi_y \cdot \Delta_x = \Delta_{\varphi_y(x)}$ for all $y \in X$ and hence $f \cdot \Delta_x = \Delta_{f(x)}$ for all $f \in \text{Inn}(X)$. Thus $f \cdot \Delta_x$ is also an equivalence class and therefore $f \cdot \Delta_x \cap \Delta_x = \emptyset$ or $f \cdot \Delta_x = \Delta_x$. This implies that $\text{Inn}(X)$ is not primitive. \square

Following [18, Definition 3.5], we say that a quandle X is of *cyclic type* (or *cyclic*) if for each $x \in X$ the permutation φ_x acts on $X \setminus \{x\}$ as a cycle of length $|X| - 1$, where $|X|$ denotes the cardinality of X . Tamaru proved that quandles of cyclic type are doubly transitive [18, Prop. 3.6]. In particular, quandles of cyclic type are indecomposable.

Example 9 (Alexander quandles). *Alexander quandles form an important family of examples. Let A be an abelian group and $g \in \text{Aut}(A)$. Then A is a quandle with $x \triangleright y = (1 - g)(x) + g(y)$ for all $x, y \in A$. This is the Alexander quandle of type (A, g) .*

Example 10. *Let us mention a particular case of Example 9. Let p be a prime number, $m \in \mathbb{N}$, $q = p^m$, and \mathbb{F}_q be the field of q elements. For each $\alpha \in \mathbb{F}_q$ the Alexander quandle of type (q, α) is the quandle structure over \mathbb{F}_q given by $x \triangleright y = (1 - \alpha)x + \alpha y$ for all $x, y \in \mathbb{F}_q$.*

2. PROOFS OF THEOREM 1 AND COROLLARY 2

Using Alexander quandles, H. Tamaru proved the existence of quandles of cyclic type with a prime number of elements, see [18, §4]. We use Tamaru's method to prove a similar result.

Recall that for any power q of a prime number, the multiplicative subgroup of \mathbb{F}_q is cyclic of order $q - 1$.

Proposition 11. *Let p be a prime number, $m \in \mathbb{N}$ and $q = p^m$. Let $\alpha \in \mathbb{F}_q$ and X be an Alexander quandle of type (q, α) . Then X is of cyclic type if and only if α has order $q - 1$.*

Proof. Suppose first that X is of cyclic type. Then φ_0 acts on $X \setminus \{0\}$ as a cycle of length $q - 1$. Thus

$$\varphi_0 = \left(1 \ \varphi_0(1) \ \varphi_0^2(1) \cdots \varphi_0^{q-2}(1)\right)$$

and $\varphi_0^i(1) \neq \varphi_0^j(1)$ for $i, j \in \{0, \dots, q - 2\}$ with $i \neq j$. Since $\varphi_0^k(1) = \alpha^k$ for all $k \in \{0, \dots, q - 2\}$, the claim follows.

Conversely, suppose that α has order $q - 1$. Since X has no nontrivial subquandles by [1, Prop. 4.1], it follows that X is indecomposable. The permutation φ_0 acts on X as the cycle $(1 \ \alpha \ \alpha^2 \cdots \alpha^{q-2})$ of length $q - 1$. Since X is indecomposable, this implies that X is of cyclic type by [18, Prop. 3.9]. \square

Now we prove that the cardinality of a finite quandle of cyclic type is some power of a prime number. For that purpose, we need some basic properties of Frobenius groups. A finite group G acting on a finite set X is a *Frobenius group* if $G_x \cap G_y = 1$ for all $x, y \in X$ with $x \neq y$, where G_x and G_y denote the stabilizer (or isotropy) subgroups of x and y respectively. The *degree* of G is the cardinality of X .

It follows from the definition that the center of a Frobenius group is trivial. The following result is a consequence of [20, Thm. 5.1] and [20, Thm. 11.3(a)].

Theorem 12. *Let G be a doubly transitive Frobenius group of degree n . Then $n = p^m$ for some prime number p and $m \in \mathbb{N}$.*

We shall also need the following two lemmas.

Lemma 13. *Let X be a finite quandle of cyclic type, $x \in X$, and $G = \text{Inn}(X)$. Then G_x is cyclic and generated by φ_x .*

Proof. Assume that X has n elements. Then G is a subgroup of \mathbb{S}_n . Since

$$f\varphi_x f^{-1} = \varphi_{f(x)} = \varphi_x$$

for all $f \in G_x$, we conclude that $G_x \subseteq C_G(\varphi_x)$, where $C_G(\varphi_x)$ denotes the centralizer of φ_x in G . The permutation φ_x is a cycle of length $n - 1$. Hence $C_G(\varphi_x) = C_{\mathbb{S}_n}(\varphi_x) \cap G = \langle \varphi_x \rangle$. Therefore $G_x = \langle \varphi_x \rangle$. \square

Lemma 14. *Let $n \geq 3$ and X be a quandle of cyclic type of size n . Then $\text{Inn}(X)$ is a Frobenius group of degree n .*

Proof. Let $G = \text{Inn}(X)$ and $x \in X$. By Lemma 13, $G_x = \langle \varphi_x \rangle$. We claim that for each $g \in G \setminus G_x$ the subgroups G_x and $gG_x g^{-1} = G_{g(x)}$ have trivial intersection. Let $h \in G_x \cap gG_x g^{-1}$ and assume that $h = g\varphi_x^k g^{-1} = \varphi_x^l$ for some $k, l \in \{0, \dots, n-2\}$ and $g \in G \setminus G_x$. Then

$$\varphi_x^l = g\varphi_x^k g^{-1} = (g\varphi_x g^{-1})^k = \varphi_{g(x)}^k.$$

Let $y \in X \setminus \{x\}$ such that $g(x) = y$. Then $\varphi_x^l = \varphi_y^k$. Since φ_y^k is a $(n-1)$ -cycle that fixes y and $\varphi_y^k(x) = \varphi_x^l(x) = x$, we conclude that $k = l = 0$. From this the claim follows. \square

Now we prove that for $n > 2$ there exists a quandle of cyclic type of size n if and only if n is a power of a prime number. This establishes [12, Conjecture 4.7].

Proof of Theorem 1. Assume that $n = p^m$, where p is a prime number and $m \in \mathbb{N}$. By Proposition 11, there exists a quandle of cyclic type of size n . Conversely, if X is a quandle of cyclic type and size n , then $\text{Inn}(X)$ is a Frobenius group by Lemma 14. Since $\text{Inn}(X)$ acts doubly transitively on X by [18, Prop. 3.6], Theorem 12 implies that n is a power of a prime number. \square

Theorem 1, Lemma 8 and the classification of simple quandles of Andraskiewitsch and Graña [2, §3] yield Corollary 2.

Proof of Corollary 2. Let us assume that X is a cyclic quandle. By Theorem 1, the cardinality of X is some power of a prime number. Since X is doubly transitive by [18, Prop. 3.6], it follows that $\text{Inn}(X)$ acts primitively on X . By Lemma 8, X is simple. Now [2, Thm. 3.9] yields the claim. \square

3. PROOF OF THEOREM 3

Recall that a *minimal normal* subgroup of G is a normal subgroup N of G such that $N \neq 1$ and N contains no normal subgroup of G except 1 and N . The *socle* of G is the product of the minimal normal subgroups of G . The following theorem goes back to Burnside, see for example [3, Thm. 4.3].

Theorem 15 (Burnside). *Let G be a doubly transitive group and N be a minimal normal subgroup of G . Then N is either a regular elementary abelian group, or a nonregular nonabelian simple group.*

Now we prove that finite doubly transitive quandles are Alexander simple. The proof uses the classification of doubly transitive groups with simple socle, see [3] and [8, § 7.7] for the details and references. Table 1 is taken from [4, Table 7.4].

TABLE 1. Doubly transitive groups with simple socle

Group	Degree	Conditions	Transitivity
\mathbb{A}_n	n	$n \geq 5$	$n - 2$
$\text{PSL}(d, q)$	$(q^d - 1)/(q - 1)$	$d \geq 2$ $(d, q) \neq (2, 2), (2, 3)$	3 if $d = 2, q$ even 2 otherwise
$\text{Sp}(2d, 2)$	$2^{2d-1} + 2^{d-1}$	$d \geq 3$	2
$\text{Sp}(2d, 2)$	$2^{2d-1} - 2^{d-1}$	$d \geq 3$	2
$\text{PSU}(3, q)$	$q^3 + 1$	$q \geq 3$	2
$\text{Sz}(q)$	$q^2 + 1$	$q = 2^{2d+1} > 2$	2
$\text{R}(q)$	$q^3 + 1$	$q = 3^{2d+1} > 3$	2
$\text{PSL}(2, 11)$	11		2
M_{11}	11		4
M_{11}	12		3
M_{12}	12		5
\mathbb{A}_7	15		2
M_{22}	22		3
M_{23}	23		4
M_{24}	24		5
$\text{PSL}(2, 8)$	28		2
HS	176		2
Co_3	276		2

Proof of Theorem 3. The quandle X is doubly transitive and hence $\text{Inn}(X)$ acts primitively on X . Then X is simple by Lemma 8 and therefore X is a conjugacy class of $\text{Inn}(X)$ by [11, Lemma 1].

We claim that $\text{Inn}(X)$ is solvable. To prove our claim, suppose that $\text{Inn}(X)$ is nonsolvable. Let N be the commutator subgroup of $\text{Inn}(X)$. Since N is the unique minimal normal subgroup of $\text{Inn}(X)$ by [11, Lemma 2] and $\text{Inn}(X)$ is nonsolvable, it follows from Theorem 15 that N is a nonabelian simple group.

Let us prove that $N = \text{Inn}(X)$. Since N is a nonabelian simple group, $Z(N) = 1$ and $[N, N] = N$. Then $N/[N, N]$ is trivial and $[N, N] = N$ is the minimal nontrivial normal subgroup of N . From [11, Prop. 3] it follows that $\text{Inn}(X) = N$.

The inner group $\text{Inn}(X)$ is permutation isomorphic to one of the groups of Table 1. Since $\text{Inn}(X)$ is not k -transitive for $k \geq 3$ by Lemma 7, the inner group $\text{Inn}(X)$ is not permutation isomorphic to \mathbb{A}_n , where $n \geq 5$, M_{11} , M_{11} , M_{12} , M_{22} , M_{23} , M_{24} . Further, $\text{Inn}(X)$ is not permutation isomorphic to the groups \mathbb{A}_7 , $\text{PSL}(2, 11)$, $\text{PSL}(2, 8)$, HS , Co_3 since these groups do not have conjugacy classes of size 15, 11, 28, 176 and 276 respectively. (This can be checked for example with the help of **GAP** and the package **atlasrep**.) Thus $\text{Inn}(X)$ is permutation isomorphic to one of the doubly transitive groups of Table 2.

We claim that none of the groups of Table 2 is permutation isomorphic to $\text{Inn}(X)$. We split the proof into several steps.

Case 1: Linear groups. Let $G = \text{PSL}(d, q)$ with $d \geq 2$ acting on the set $\mathbb{P}(V)$ of projective lines of a d -dimensional vector space V over the field \mathbb{F}_q . This set has size $(q^d - 1)/(q - 1)$. To prove that the doubly transitive group G is not permutation

TABLE 2. Some doubly transitive groups with simple socle.

Group	Degree	Conditions
$\text{PSL}(d, q)$	$\frac{q^d - 1}{q - 1}$	$d \geq 2$
$\text{Sp}(2d, 2)$	$2^{2d-1} + 2^{d-1}$	$d \geq 3$
$\text{Sp}(2d, 2)$	$2^{2d-1} - 2^{d-1}$	$d \geq 3$
$\text{PSU}(3, q)$	$q^3 + 1$	$q \geq 3$
$\text{Sz}(q)$	$q^2 + 1$	$q = 2^{2d+1} > 2$
$\text{R}(q)$	$q^3 + 1$	$q = 3^{2d+1} > 3$

isomorphic to $\text{Inn}(X)$ we use Lemma 6: Let $[v] \in \mathbb{P}(V)$ and denote by $G_{[v]}$ the stabilizer of $[v]$ in G . An elementary calculation shows that $Z(G_{[v]}) = 1$. Hence G and $\text{Inn}(X)$ are not permutation isomorphic.

Case 2: Symplectic groups. Let $G = \text{Sp}(2d, 2)$ with $d \geq 3$. There are two doubly transitive actions on sets Ω of size $2^{d-1}(2^d + 1)$ and $2^{d-1}(2^d - 1)$. The stabilizers G_α are primitive on $\Omega \setminus \{\alpha\}$. Since $\langle \varphi_x \rangle$ is a nontrivial central subgroup of G_α , it follows from [20, Thm. 8.8] that $\langle \varphi_x \rangle$ acts transitively on $X \setminus \{x\}$. Thus X is of cyclic type by [18, Prop. 3.9]. Since the quandle is some power of a prime number by Theorem 1, G has a conjugacy class of prime power size. Since nonabelian simple groups do not have nontrivial conjugacy classes with prime power size [10, Thm. 3.9], it follows that G and $\text{Inn}(X)$ are not permutation isomorphic.

Case 3: Unitary groups. Let $G = \text{PSU}(3, q)$ with $q \geq 3$ with the natural doubly transitive action on a set Ω of size $q^3 + 1$. By inspection of [16, Table 2], the centralizers of elements of G have sizes:

$$\begin{aligned}
 & q^3(q+1)/d, & q^2, \\
 & q(q+1)^2(q-1)/d, & q(q+1)/d, \\
 & (q+1)^2, & (q+1)^2/d, \\
 & (q^2 - q + 1)/d, & q^3(q^3 + 1)(q^2 - 1)/d,
 \end{aligned}
 \tag{3.1}$$

where $d = \gcd(3, q+1)$ is the greatest common divisor of 3 and $q+1$. (Recall that the order of G is $q^3(q^3 + 1)(q^2 - 1)/d$.) An easy calculation then shows that G has no conjugacy classes of size $q^3 + 1$. Since the quandle X is a conjugacy class of $\text{Inn}(X)$, it follows that G and $\text{Inn}(X)$ are not permutation isomorphic.

Case 4: Suzuki groups. Let $G = \text{Sz}(q) = {}^2B_2(q)$ with $q^{2d+1} > 2$ with the natural doubly transitive action on a set of size $q^2 + 1$. Suzuki groups are (ZT)-groups and hence the stabilizers have trivial center. (To prove our claim we follow Suzuki's argument [17, page 107]: Since $\text{Sz}(q)$ is a (ZT)-group, the identity is the only permutation fixing three points. Then stabilizers are Frobenius groups and Frobenius groups have trivial centers.) From Lemma 6 we conclude that G is not permutation isomorphic to $\text{Inn}(X)$.

Case 5: Ree groups. Let $G = \text{R}(q) = {}^2G_2(q)$ with $q = 3^{2d+1} > 3$ with the natural doubly transitive action on a set Ω of size $q^3 + 1$. By [21, §4.5.2], the stabilizer G_α of $\alpha \in \Omega$ is isomorphic to the subgroup of $\text{SL}(7, q)$ generated by the diagonal matrices

$$d(\lambda) = \text{diag}(\lambda, \lambda^{3^{d+1}-1}, \lambda^{-3^{d+1}+2}, 1, \lambda^{3^{d+1}-2}, \lambda^{-3^{d+1}+1}, \lambda^{-1}),$$

where $\lambda \in \mathbb{F}_q \setminus \{0\}$, and the matrices

$$a = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

A direct calculation shows that the center of G_α is trivial. Then Lemma 6 implies that G is not permutation isomorphic to $\text{Inn}(X)$.

The permutation groups of Table 1 are not permutation isomorphic to $\text{Inn}(X)$. Hence $\text{Inn}(X)$ is solvable. Since X is simple and $\text{Inn}(X)$ is solvable, there exists a prime number p and $m \in \mathbb{N}$ such that X is an Alexander quandle of size p^m by [2, Thm. 3.9]. \square

Acknowledgement. The author thanks E. Clark, S. Kamada, A. Lochmann, J. McCarron, H. Tamaru and K. Wada for several helpful comments. The referee deserves thanks for careful reading and many useful comments. This work is supported by Conicet, UBACyT 20020110300037, ICTP, and the Alexander von Humboldt Foundation.

REFERENCES

- [1] N. Andruskiewitsch, F. Fantino, G. A. García, and L. Vendramin. On Nichols algebras associated to simple racks. In *Groups, algebras and applications*, volume 537 of *Contemp. Math.*, pages 31–56. Amer. Math. Soc., Providence, RI, 2011.
- [2] N. Andruskiewitsch and M. Graña. From racks to pointed Hopf algebras. *Adv. Math.*, 178(2):177–243, 2003.
- [3] P. J. Cameron. Finite permutation groups and finite simple groups. *Bull. London Math. Soc.*, 13(1):1–22, 1981.
- [4] P. J. Cameron. *Permutation groups*, volume 45 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 1999.
- [5] J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford, and M. Saito. Quandle cohomology and state-sum invariants of knotted curves and surfaces. *Trans. Amer. Math. Soc.*, 355(10):3947–3989, 2003.
- [6] W. E. Clark, M. Elhamdadi, X.-d. Hou, M. Saito, and T. Yeatman. Connected quandles associated with pointed abelian groups. *Pacific J. Math.*, 264(1):31–60, 2013.
- [7] W. E. Clark, M. Elhamdadi, M. Saito, and T. Yeatman. Quandle Colorings of Knots and Applications. *arXiv:1312.3307*.
- [8] J. D. Dixon and B. Mortimer. *Permutation groups*, volume 163 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996.
- [9] C. Hayashi. Canonical forms for operation tables of finite connected quandles. *Comm. Algebra*, 41(9):3340–3349, 2013.
- [10] I. M. Isaacs. *Character theory of finite groups*. AMS Chelsea Publishing, Providence, RI, 2006. Corrected reprint of the 1976 original [Academic Press, New York; MR0460423].
- [11] D. Joyce. Simple quandles. *J. Algebra*, 79(2):307–318, 1982.
- [12] S. Kamada, H. Tamaru, and K. Wada. On classification of quandles of cyclic type. *arXiv:1312.6917*.
- [13] P. Lopes and D. Roseman. On finite racks and quandles. *Comm. Algebra*, 34(1):371–406, 2006.
- [14] J. McCarron. Connected quandles with order equal to twice an odd prime. *arXiv:1210.2150*.
- [15] J. McCarron. Small homogeneous quandles. In *Proceedings of the 37th International Symposium on Symbolic and Algebraic Computation*, ISSAC ’12, pages 257–264, New York, NY, USA, 2012. ACM.

- [16] W. A. Simpson and J. S. Frame. The character tables for $SL(3, q)$, $SU(3, q^2)$, $PSL(3, q)$, $PSU(3, q^2)$. *Canad. J. Math.*, 25:486–494, 1973.
- [17] M. Suzuki. On a class of doubly transitive groups. *Ann. of Math. (2)*, 75:105–145, 1962.
- [18] H. Tamaru. Two-point homogeneous quandles with prime cardinality. *J. Math. Soc. Japan*, 65(4):1117–1134, 2013.
- [19] K. Wada. Two-point homogeneous quandles with cardinality of prime power. *Preprint*.
- [20] H. Wielandt. *Finite permutation groups*. Translated from the German by R. Bercov. Academic Press, New York, 1964.
- [21] R. A. Wilson. *The finite simple groups*, volume 251 of *Graduate Texts in Mathematics*. Springer-Verlag London Ltd., London, 2009.

DEPARTAMENTO DE MATEMÁTICA – FCEN, UNIVERSIDAD DE BUENOS AIRES, PAB. I – CIUDAD
UNIVERSITARIA (1428) BUENOS AIRES – ARGENTINA
E-mail address: `lvendramin@dm.uba.ar`